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TOTAL MARKS

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NATIONAL SENIOR CERTIFICATE EXAMINATION  
MAY 2023

**MATHEMATICS: PAPER II**

EXAMINATION NUMBER

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Time: 3 hours

150 marks

**PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY**

1. This question paper consists of 32 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand it in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Ensure that your calculator is in **DEGREE** mode.
7. Clearly show **ALL** calculations, diagrams, graphs, etc. that you have used in determining your answers. **Answers only will NOT necessarily be awarded full marks.**
8. Round off to **ONE DECIMAL PLACE** unless otherwise stated.
9. It is in your own interest to write legibly and to present your work neatly.
10. **FOUR** blank pages (pages 29–32) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

**FOR OFFICE USE ONLY: MARKER TO ENTER MARKS**

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	TOTAL
10	10	15	10	11	8	6	10	5	16	16	13	8	12	/150

**SECTION A****QUESTION 1**

The data below gives the year marks (out of 100), obtained by 8 pupils in Mathematics and Physical Science.

Maths Mark ( $x$ )	67	77	26	99	49	38	56	58
Physical Science Mark ( $y$ )	75	83	32	94	61	44	65	68

- (a) Use your calculator to determine the equation for the line of best fit in the form  $y = A + Bx$ . **Give your answer correct to 3 decimal places.**

(3)

- (b) (1) Calculate the correlation coefficient for the data.

(1)

- (2) What does the correlation coefficient suggest about the relationship between the mathematics year mark and the physical science year mark?

(2)

(c) Khaya attained 70 for his mathematics year mark. Use the model in (a) to predict his physical science year mark.

(2)

(d) Comment on the accuracy of the prediction in (c).

(2)  
**[10]**

**QUESTION 2**

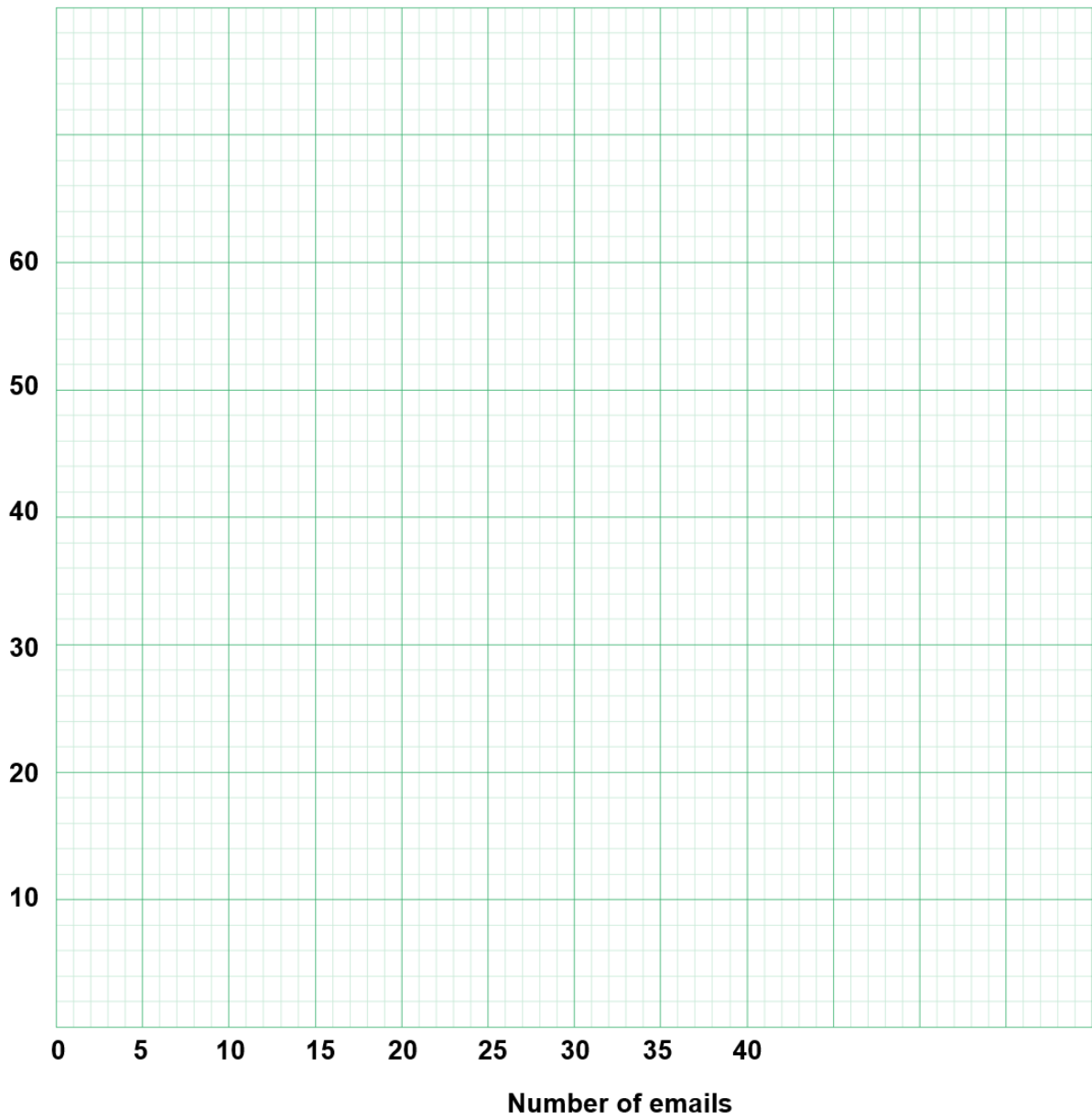
The number of emails received by an individual daily over a 50-day period is illustrated in the table.

<b>Number of emails (<math>x</math>)</b>	<b>Frequency</b>
$0 < x \leq 10$	13
$10 < x \leq 20$	21
$20 < x \leq 30$	12
$30 < x \leq 40$	4

- (a) Determine the estimated mean of the data.

(3)

(b) Represent the data on a cumulative frequency curve (ogive).



(5)

- (c) Using your answer to (a) and by determining the median as well, would you describe the data as **positively** or **negatively** skewed? Explain.

(2)  
**[10]**

**QUESTION 3**

(a) Simplify without using a calculator:

$$\sin(85^\circ + \theta) \cdot \cos(25^\circ + \theta) - \cos(85^\circ + \theta) \cdot \sin(25^\circ + \theta)$$

(2)

(b) Simplify:  $\tan(-\theta) \cdot \sin(90^\circ + \theta) + \frac{\sin 2\theta}{2\cos(360^\circ + \theta)}$

(6)

(c) Given:  $\cos x = p$  and  $0^\circ < x < 180^\circ$   
Express each of the following in terms of  $p$ :

(1)  $\sin x$

(2)

(2)  $\cos 2x$

(2)

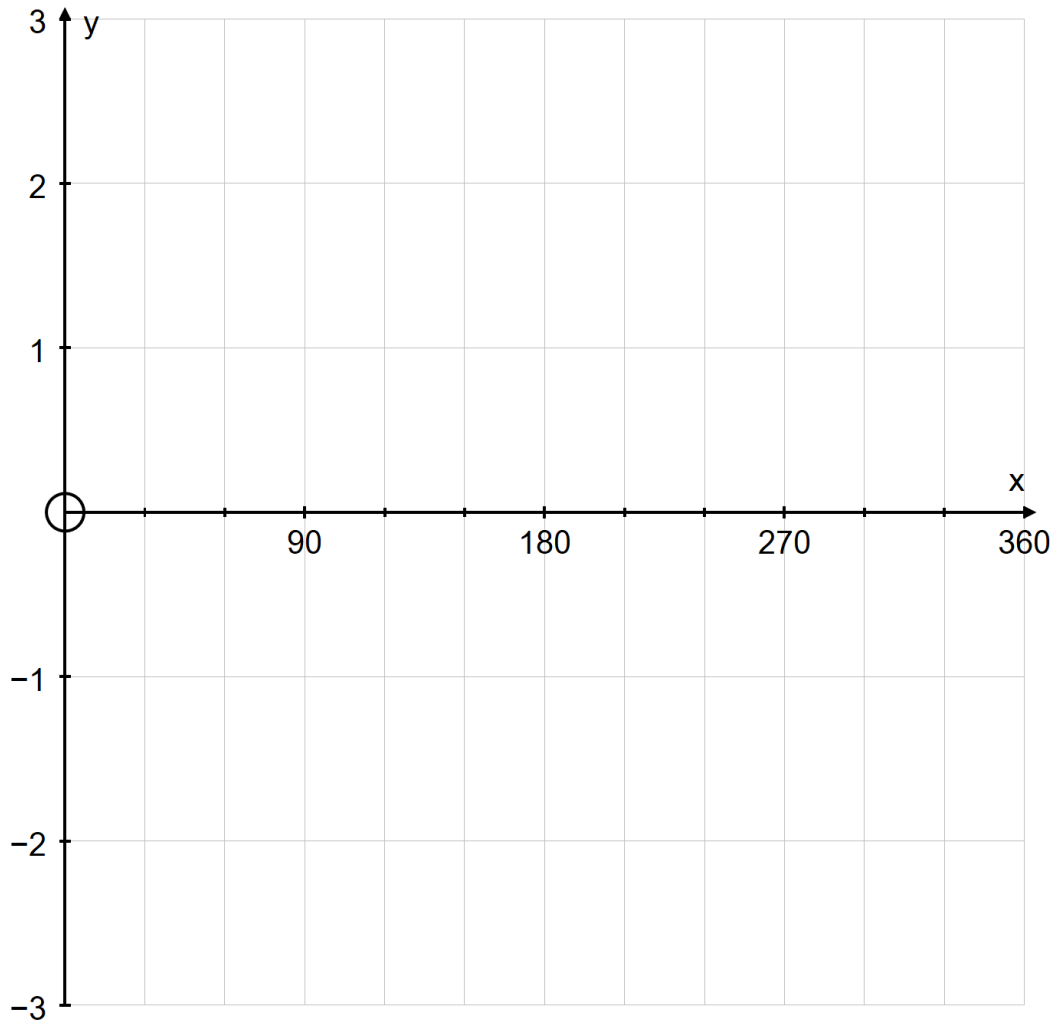
(d) Determine the general solution for:  $2\sin(x - 30^\circ) = \cos(x - 30^\circ)$

(3)  
**[15]**

**QUESTION 4**

Given:  $f(x) = 2\sin x$  and  $g(x) = \cos(x + 30^\circ)$  for  $x \in [0^\circ; 360^\circ]$

- (a) Sketch the graphs of  $f$  and  $g$  on the same set of axes below.  
Label all intercepts with the axes, turning point and end points.



(6)

(b) Show that the graph of  $g$  can be written as  $g(x) = \frac{1}{2}(\sqrt{3} \cos x - \sin x)$ .

(2)

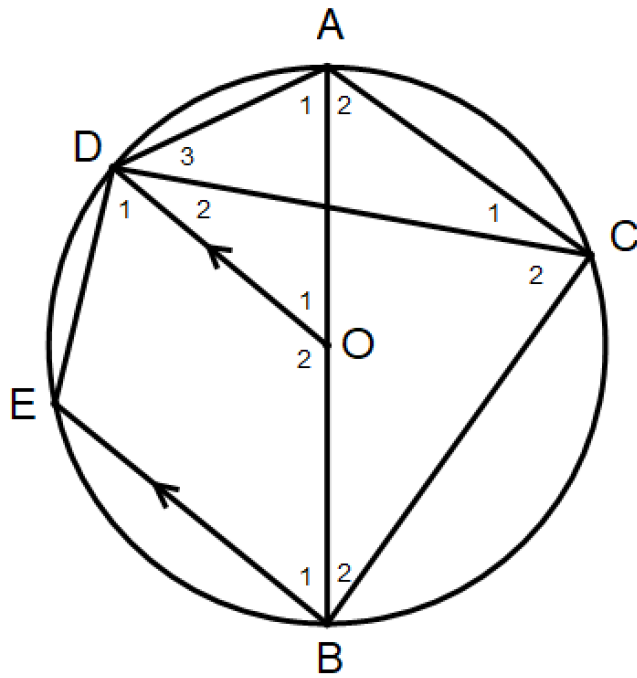
(c) Indicate clearly on your graph, the values of  $x$  for which  $4 \sin x = \sqrt{3} \cos x - \sin x$ .

(2)  
**[10]**

**QUESTION 5**

In the diagram:

- AB is the diameter of the circle with centre O.
- D and E are points on the circle so that  $DO \parallel EB$
- C is a point on the circle as shown.
- $\hat{O}_1 = 40^\circ$



**State all relevant reasons with your statements.**

(a) Determine:  $\hat{A}_1$

(2)

(b) Determine:  $\hat{E}$

(2)

(c) Determine:  $\hat{C}_1$

(2)

(d) Determine:  $\hat{B}_1$

(1)

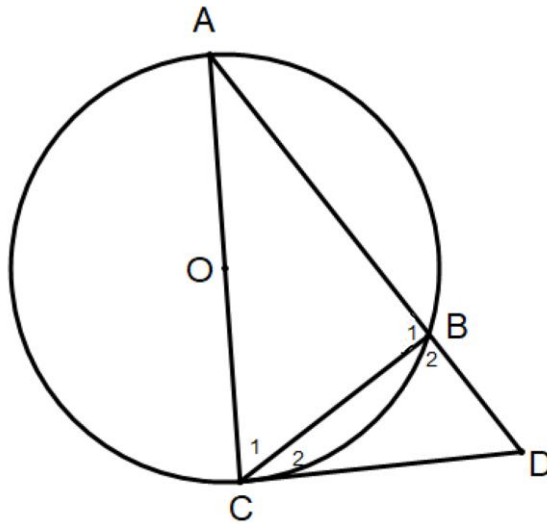
(e) If AE is constructed and intersects DO at F, determine the length of AE if the diameter of the circle is 9 units.

(4)  
**[11]**

**QUESTION 6**

In the diagram:

- AC is a diameter of the circle with centre O.
- B is a point on the circle and AB is produced to D so that DC is a tangent to the circle at C.
- AB = 5 units
- BD = 4 units
- $\hat{C}_2 = x$



(a) Determine  $\hat{A}$  in terms of  $x$ .

(2)

(b) Prove that  $\triangle CBD \sim \triangle ACD$

(3)

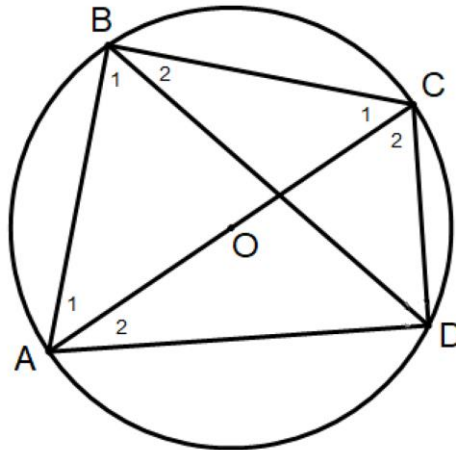
(c) Determine the length of the tangent CD.

(3)  
[8]

**QUESTION 7**

In the diagram:

- AC is a diameter of the circle with centre O.
- BD is a chord of the circle.
- $\hat{B}_1 = 67,5^\circ$
- $\hat{C}_1 = 45^\circ$



(a) Is  $\triangle ABC$  an isosceles triangle? Explain.

(3)

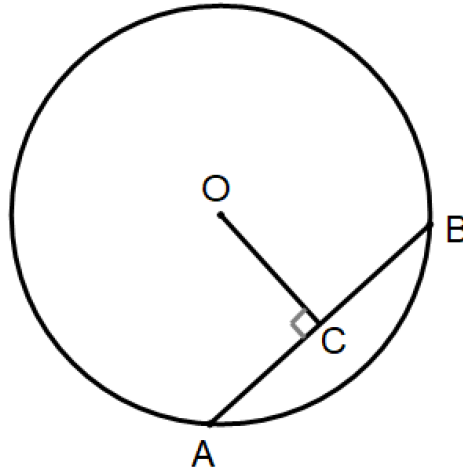
(b) Prove:  $\hat{A}_1 = 2 \times \hat{A}_2$

(3)  
[6]

**70 marks**

**SECTION B****QUESTION 8**

- (a) Use the diagram below to prove the theorem that **a line drawn from the centre of a circle, perpendicular to a chord, bisects the chord.**



Required to prove:  $AC = CB$

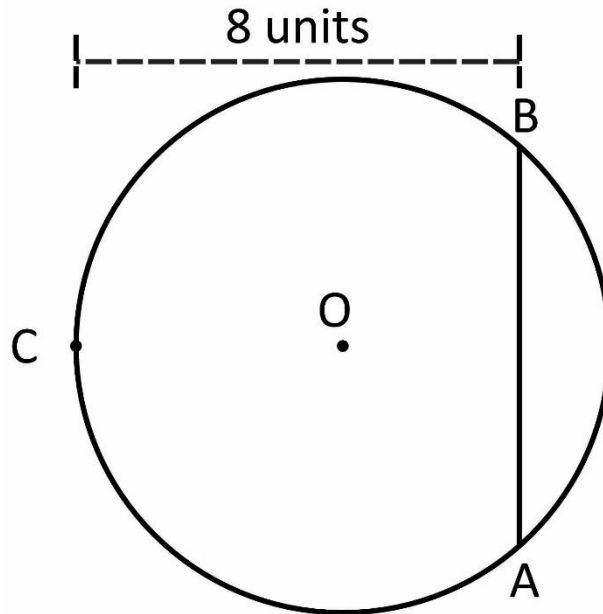
Construction:

Proof:

(5)

(b) In the diagram:

- O is the centre of the circle.
- Chord AB = 8 units.
- The perpendicular distance from the furthest point C on the circle to chord AB is 8 units.



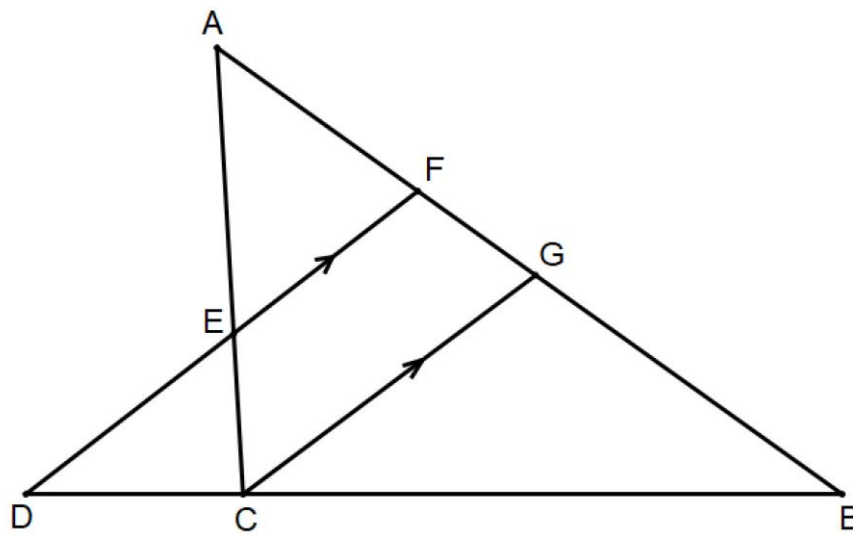
Determine the radius of the circle with centre O.

(5)  
[10]

**QUESTION 9**

In the diagram  $\triangle ABC$  is drawn.

- E and F are points on AC and AB respectively such that  $\frac{AE}{EC} = \frac{3}{2}$  and  $\frac{AF}{FB} = \frac{2}{5}$
- BC produced meets FE produced in D.
- G is a point on FB so that  $FD \parallel GC$ .



Calculate, giving reasons:  $\frac{BC}{CD}$

**QUESTION 10**

- (a) (1) If  $f$  is a single trigonometric function and  $\frac{\sin\theta \cos 2\theta}{\sin 2\theta} \div \left( \tan\theta - \frac{1}{\tan\theta} \right) = af(\theta)$ , determine the constant  $a$  and the trigonometric function  $f(\theta)$ .

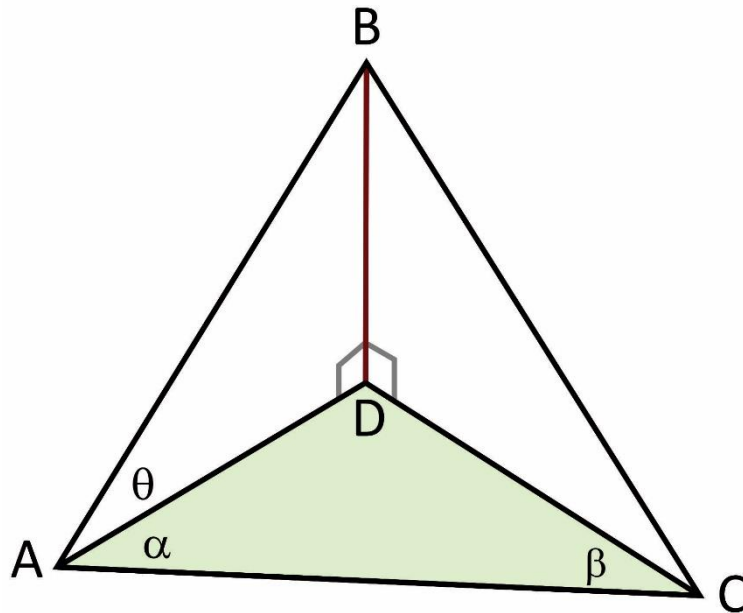
(6)

- (2) Determine the values of  $\theta \in [0^\circ; 360^\circ]$  for which the identity in (a)(1) is not valid.

(5)

(b) In the diagram below:

- ADC represents a horizontal triangular field.
- BD represents a vertical building standing on ADC at D.
- $AC = 120$  metres.
- $\hat{D}AC = \alpha$
- $\hat{A}CD = \beta$
- The angle of elevation of B from A is  $\theta$ .



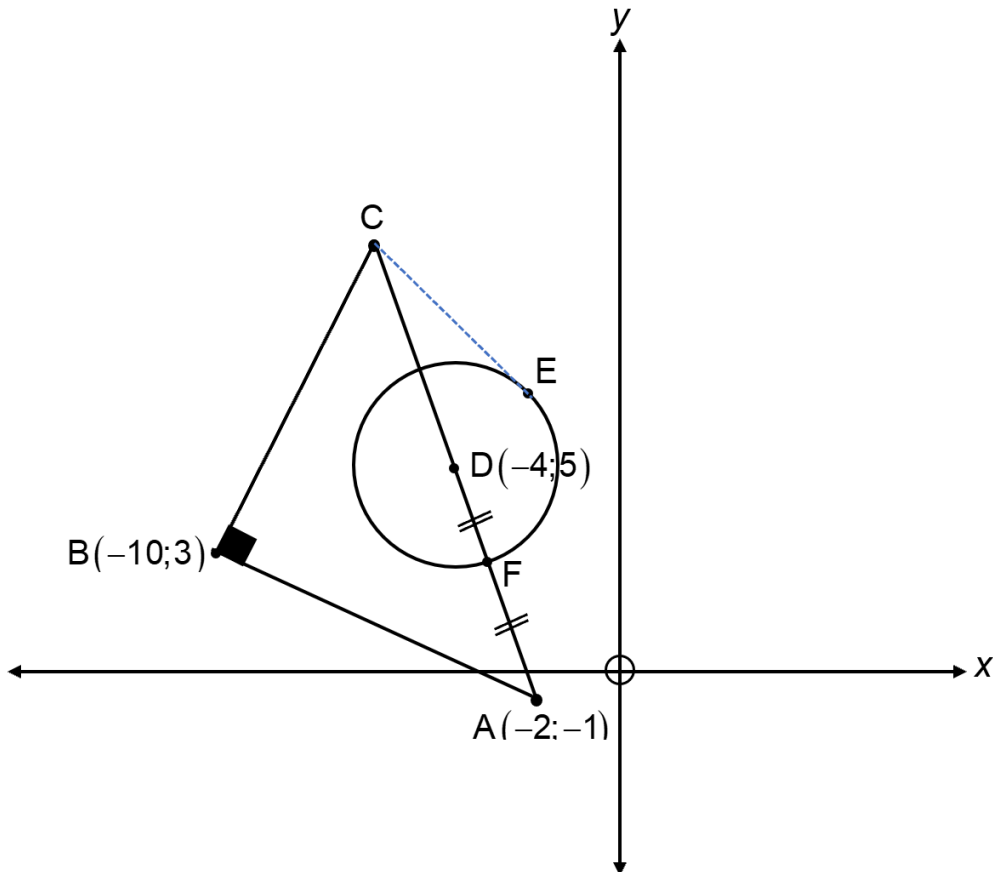
Determine the height of the building BD in terms of  $\alpha$ ,  $\beta$  and  $\theta$ . Simplify your answer.

(5)  
[16]

**QUESTION 11**

In the diagram,  $\triangle ABC$  is drawn with  $A(-2;-1)$  and  $B(-10;3)$ .

- $D(-4;5)$  is the centre of the circle.
- $ADC$  cuts the circle at  $F$  with  $AF = FD$ .
- $E$  is a point on the circle so that  $CE$  is a tangent to the circle at  $E$ .
- $BC \perp AB$

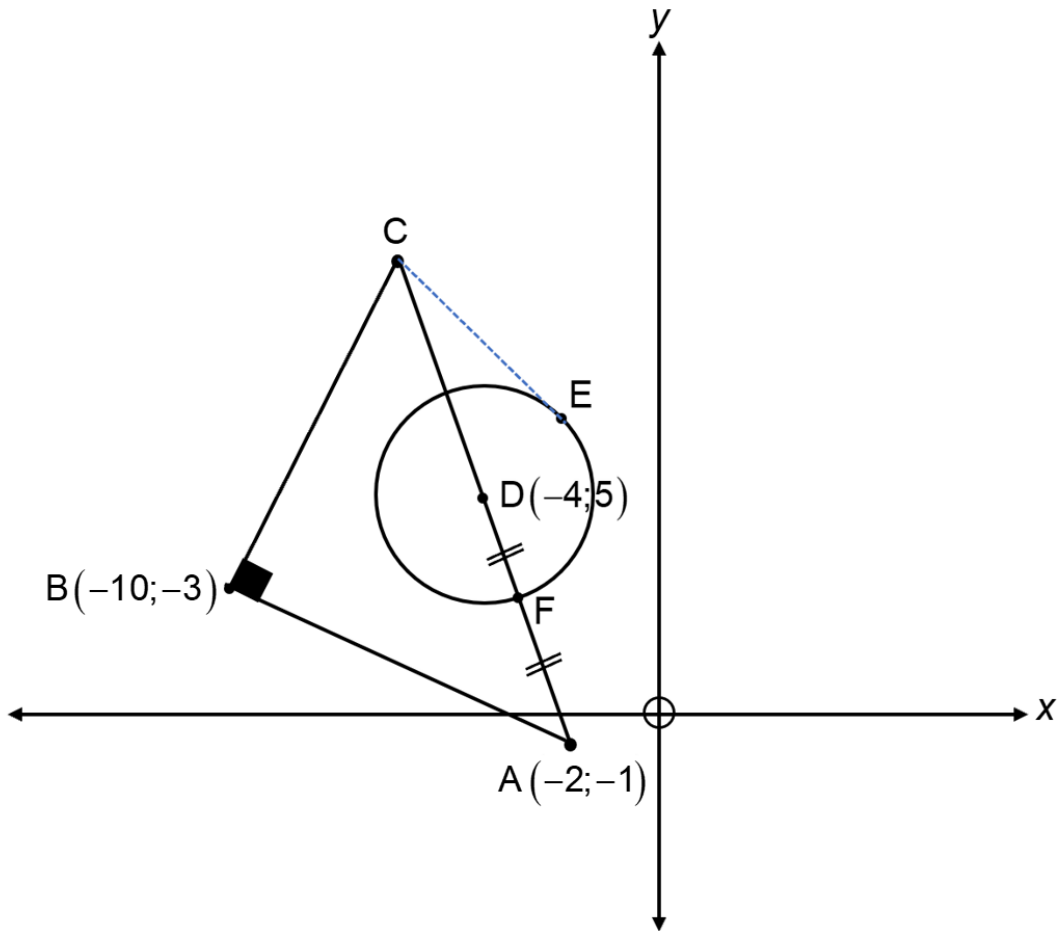


(a) Determine the equation of the circle with centre  $D$ .

(5)

(b) Determine the coordinates of C.

(7)



- (c) Determine the length of the tangent CE if it is given that the coordinates of C is  $(-6;11)$ .

(4)  
[16]

**QUESTION 12**

The equation of the circle with centre P is given as  $x^2 - 14x + y^2 - 2y = -46$ .

- (a) Determine the centre and radius of the circle with centre P.

(3)

- (b) Point A  $(6; \sqrt{3} + 1)$  lies on the circle. Determine the equation of the tangent to the circle at A.

(4)

- (c) Consider the point B  $(8; -\sqrt{3} + 1)$ .

- (1) Does point B lie on the circle? Explain.

(3)

- (2) Is AB the diameter of the circle? Explain.

(3)  
**[13]**

**QUESTION 13**

In Diagram 1 below, ABCD is the rectangular base of one section of the roof of a hut in the mountains in Lesotho. The isosceles triangle BEC will form another section of the roof.

- $AB = 12\text{ cm}$
- $AD = 8\text{ cm}$
- $EM = 6\text{ cm}$
- $BE = CE$

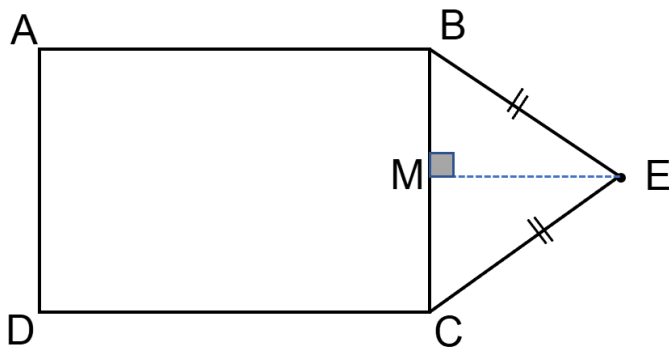
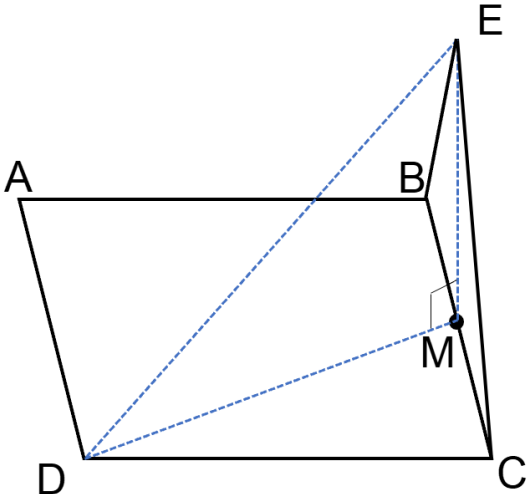


Diagram 1

In Diagram 2 below,  $\triangle BEC$  from Diagram 1 is folded up along BC such that it is perpendicular to the base ABCD.

Diagram 2



- (a) When the roof is completed,  $\hat{BEC}$  needs to be smaller than  $65^\circ$  to prevent snow building up on the roof. Determine whether or not snow will build up on the roof.

(3)

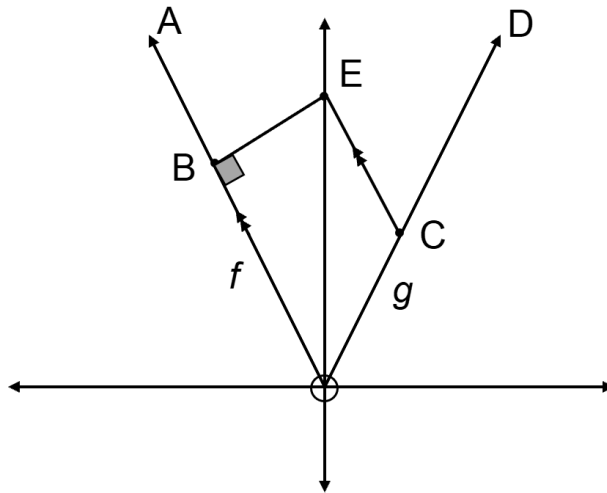
(b) Determine  $\hat{D}EC$  as shown in Diagram 2.

(5)  
[8]

**QUESTION 14**

In the diagram:

- O represents the origin.
- A is a point on the line with equation  $f(x) = -2x$ .
- B is a point on OA so that  $OB = \sqrt{125}$  units.
- E is a point on the y axis so that EB is perpendicular to OA.
- D is a point on the line with equation  $g(x) = 2x$ .
- C is a point on OD so that EC is parallel to OA.



(a) Determine the coordinates of B.

(4)

(b) If  $B(-5;10)$  is given, determine the area of the trapezium OBEC.

(8)  
[12]

**80 marks**

**Total: 150 marks**

**ADDITIONAL SPACE (ALL QUESTIONS)**

**REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.**





